

The Light Quark Masses with the Wilson Quark Action using Chiral Ward Identities*

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We present results for the light quark masses for the Wilson quark action obtained with the PCAC relation for the one-link extended axial vector current in quenched QCD at $\beta = 5.9 - 6.5$. This method leads to a remarkable improvement of scaling behavior of the light quark masses compared to the conventional method. We obtain $\overline{m}_l = 3.87(37)\text{MeV}$ for the averaged up and down quark mass and $\overline{m}_s = 97(9)\text{MeV}$ for the strange quark mass in the $\overline{\text{MS}}$ scheme at $\mu = 2\text{GeV}$.

1. Introduction

The chiral symmetry breaking term in the Wilson quark action causes large scaling violation effects of $O(a)$ for physical quantities in numerical simulations of lattice QCD. A manifestation of this effect is that the light quark masses for the Wilson action show a strong a dependence at finite lattice spacings in contrast to a weak dependence for the case with the Kogut-Susskind (KS) action which retains U(1) chiral symmetry[1]. Uncertainties in a long extrapolation to the continuum limit needed for the Wilson case forms a part of the difficulty to settle the question whether the Wilson and KS quark actions give a consistent result in the continuum limit[2]. This leads us to reconsider the definition of quark mass for the Wilson action.

It is well known that the current quark mass defined by the PCAC relation for the Wilson

action[3,4] differs by $O(a)$ from the conventionally defined quark mass at finite lattice spacing. We examine this point in more detail and find that the former definition applied with the one-link separated axial vector current that naturally arises in chiral Ward identities[3] leads to a significantly improved scaling behavior for the light quark masses.

Our calculations are carried out as a part of our simulation for the kaon B -parameter[5] with the Wilson quark action in quenched QCD. The plaquette gauge action is employed, and data are collected at $\beta = 5.9 - 6.5$. Point source quark propagators calculated for four values of the hopping parameter at each β are used for this work. Errors are estimated by the single elimination jackknife procedure for all measured quantities.

2. Calculational method

The conventional(CV) definition of quark mass is given by $m_q^{\text{CV}} a = (1/K - 1/K_c^{\text{CV}})/2$ where

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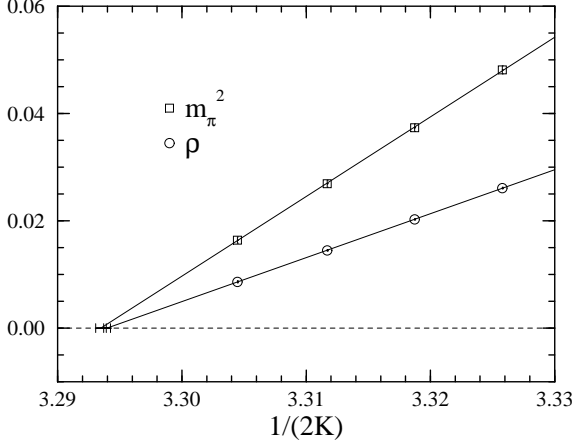


Figure 1. m_π^2 and ρ as a function of $1/(2K)$ at $\beta = 6.3$.

the critical hopping parameter K_c^{CV} is determined by extrapolating the pseudo scalar meson mass squared m_{PS}^2 linearly to $m_{PS}^2 = 0$ in the inverse hopping parameter $1/(2K)$. To extract the averaged up and down quark mass $m_l^{\text{CV}} = (m_u^{\text{CV}} + m_d^{\text{CV}})/2$ and the strange quark mass m_s^{CV} we assume the following quark mass dependence for m_{PS} and the vector meson mass m_V ,

$$m_{PS}^2 = C_{PS}(m_{q1}^{\text{CV}} + m_{q2}^{\text{CV}})/2 \quad (1)$$

$$m_V = m_V^0 + C_V(m_{q1}^{\text{CV}} + m_{q2}^{\text{CV}})/2. \quad (2)$$

The lattice spacing a is fixed with m_V^0 using $m_\rho = 770\text{MeV}$. We employ the hadron mass ratio $m_\pi/m_\rho = 0.1783$ to determine the averaged up and down quark mass m_l^{CV} . The strange quark mass m_s^{CV} is estimated in two ways using $m_K/m_\rho = 0.644$ and $m_\phi/m_\rho = 1.323$. We convert m_q^{CV} into \bar{m}_q^{CV} defined in the $\overline{\text{MS}}$ scheme at the scale $\mu = 1/a$ using the tadpole-improved perturbative mass renormalization factor $Z_m(\mu a = 1)$ evaluated with $\alpha_{\overline{\text{MS}}}(1/a)$.

The Ward identity (WI) method requires a calculation of the ρ parameter defined by the PCAC relation[3];

$$2\rho(K) = \frac{\langle 0 | \nabla_\mu A_\mu^{\text{ext},a} | \pi^a(\mathbf{p} = 0) \rangle}{\langle 0 | P^a | \pi^a(\mathbf{p} = 0) \rangle}, \quad (3)$$

We take the one-link extended axial vector current $A_\mu^{\text{ext},a}$ since it is this current

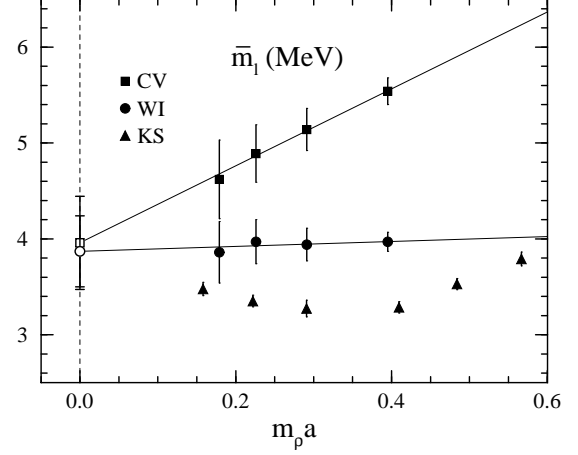


Figure 2. Averaged up and down quark mass in the $\overline{\text{MS}}$ scheme at $\mu = 2\text{GeV}$ as a function of a .

which naturally arises in the Ward identities. We extract the ρ parameter from the ratio $\langle \nabla_4 A_4^{\text{ext}}(t) P(1) \rangle / \langle P(t) P(1) \rangle / 4 + \langle \nabla_4 A_4^{\text{ext}}(T-t+2) P(1) \rangle / \langle P(T-t+2) P(1) \rangle / 4$, each two-point function projected to the zero spatial momentum, by fitting a plateau as a function of t .

In Fig. 1 a representative result for the ρ parameter is plotted as a function of $1/(2K)$ together with m_π^2 . We observe a clear linear behavior both for ρ and m_π^2 . The critical hopping parameter K_c^{WI} extracted from a linear extrapolation for ρ is slightly different from K_c^{CV} for m_π^2 (see Table 1 for numerical details). We ascribe the discrepancy to uncertainties in the extrapolations of ρ and m_π^2 , because the critical hopping parameter obtained with the two definitions should agree at each β . To avoid this problem, we define the bare quark mass for the WI method $m_q^{\text{WI}}(1/a)$ by

$$m_q^{\text{WI}}(1/a) = \frac{\rho(K)}{(1/K - 1/K_c^{\text{WI}})/2} m_q^{\text{CV}}(1/a). \quad (4)$$

We convert quark masses defined on the lattice into those in the $\overline{\text{MS}}$ scheme at the scale $\mu = 1/a$ using the tadpole-improved perturbative renormalization factor $Z_{A^{\text{ext}}}(\mu a = 1)/Z_P(\mu a = 1)$ evaluated with $\alpha_{\overline{\text{MS}}}(1/a)$.

We quote final results for $\bar{m}_q^{\text{CV,WI}}$ at the scale $\mu = 2\text{GeV}$ which are obtained by a two-loop renormalization group running from $\mu = 1/a$ [6].

Table 1

Critical hopping parameters and the light quark masses in the $\overline{\text{MS}}$ scheme at $\mu = 2\text{GeV}$.

| | input | $\beta=5.9$ | 6.1 | 6.3 | 6.5 | $a = 0$ |
|------------------------------------|-----------------|-------------|------------|------------|------------|----------|
| K_c^{CV} | | 0.15986(3) | 0.15502(2) | 0.15182(2) | 0.14946(3) | |
| K_c^{WI} | | 0.15976(1) | 0.15497(1) | 0.15179(1) | 0.14948(1) | |
| $\overline{m}_l^{\text{CV}}$ (MeV) | m_π/m_ρ | 5.54(14) | 5.14(22) | 4.89(30) | 4.62(41) | 3.96(49) |
| $\overline{m}_l^{\text{WI}}$ (MeV) | m_π/m_ρ | 3.97(10) | 3.94(17) | 3.97(23) | 3.86(32) | 3.87(37) |
| $\overline{m}_s^{\text{CV}}$ (MeV) | m_K/m_ρ | 139(3) | 129(5) | 123(8) | 116(10) | 100(12) |
| $\overline{m}_s^{\text{WI}}$ (MeV) | m_K/m_ρ | 100(3) | 99(4) | 100(6) | 97(8) | 97(9) |
| $\overline{m}_s^{\text{CV}}$ (MeV) | m_ϕ/m_ρ | 184(13) | 149(17) | 151(24) | 120(23) | |
| $\overline{m}_s^{\text{WI}}$ (MeV) | m_ϕ/m_ρ | 132(9) | 115(13) | 122(20) | 99(19) | |

3. Results for light quark masses

In Table 1 we summarize the results for the averaged up and down quark mass \overline{m}_l . The lattice spacing dependence of \overline{m}_l is shown in Fig. 2. The results with the WI method show a remarkably flat behavior. This allows a reliable extrapolation to the continuum limit by a linear function in a , with which we find $\overline{m}_l^{\text{WI}} = 3.87(37)\text{MeV}$ at $a = 0$. In contrast a large scaling violation effect is seen for the results with the CV method. While a linear extrapolation yields a consistent result $\overline{m}_q^{\text{CV}} = 3.96(49)\text{MeV}$ in the continuum limit, systematic uncertainties due to a long extrapolation are quite large. Finding a theoretical explanation as to why finite a corrections are so small for quark masses with the WI method would be an interesting problem[7].

In Fig. 2 we also plot the KS results[8] for comparison. They show small scaling violations and are systematically smaller than the results for the WI method. However, we cannot conclude at this stage whether the Wilson results are inconsistent with the KS results. The one-loop correction in the mass renormalization factor for the KS quark action is 50% – 100% depending on the lattice spacing for the results in Fig. 2, which suggests that higher order corrections might be large.

Our results for the strange quark mass \overline{m}_s determined from m_K and m_ϕ are listed in Table 1. We find that results with the WI method are quite flat also in this case.

Estimates of \overline{m}_s using m_K are expected to satisfy $\overline{m}_s \approx 25\overline{m}_l$ for each β if we assume the functional form (1), since then $m_K^2/m_\pi^2 =$

$(\overline{m}_l + \overline{m}_s)/(2\overline{m}_l) \approx 13$. Making linear extrapolations in a for the results of the WI and the CV method we find a mutually consistent result in the continuum limit: $\overline{m}_s^{\text{WI}} = 97(9)\text{MeV}$ and $\overline{m}_s^{\text{CV}} = 100(12)\text{MeV}$.

For the alternative determination of \overline{m}_s with the use of m_ϕ , the discrepancy between the WI and the CV results reduces toward the continuum limit. However, large errors originating from those of the vector meson mass hinder us from reliably extrapolating the results to the continuum limit.

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